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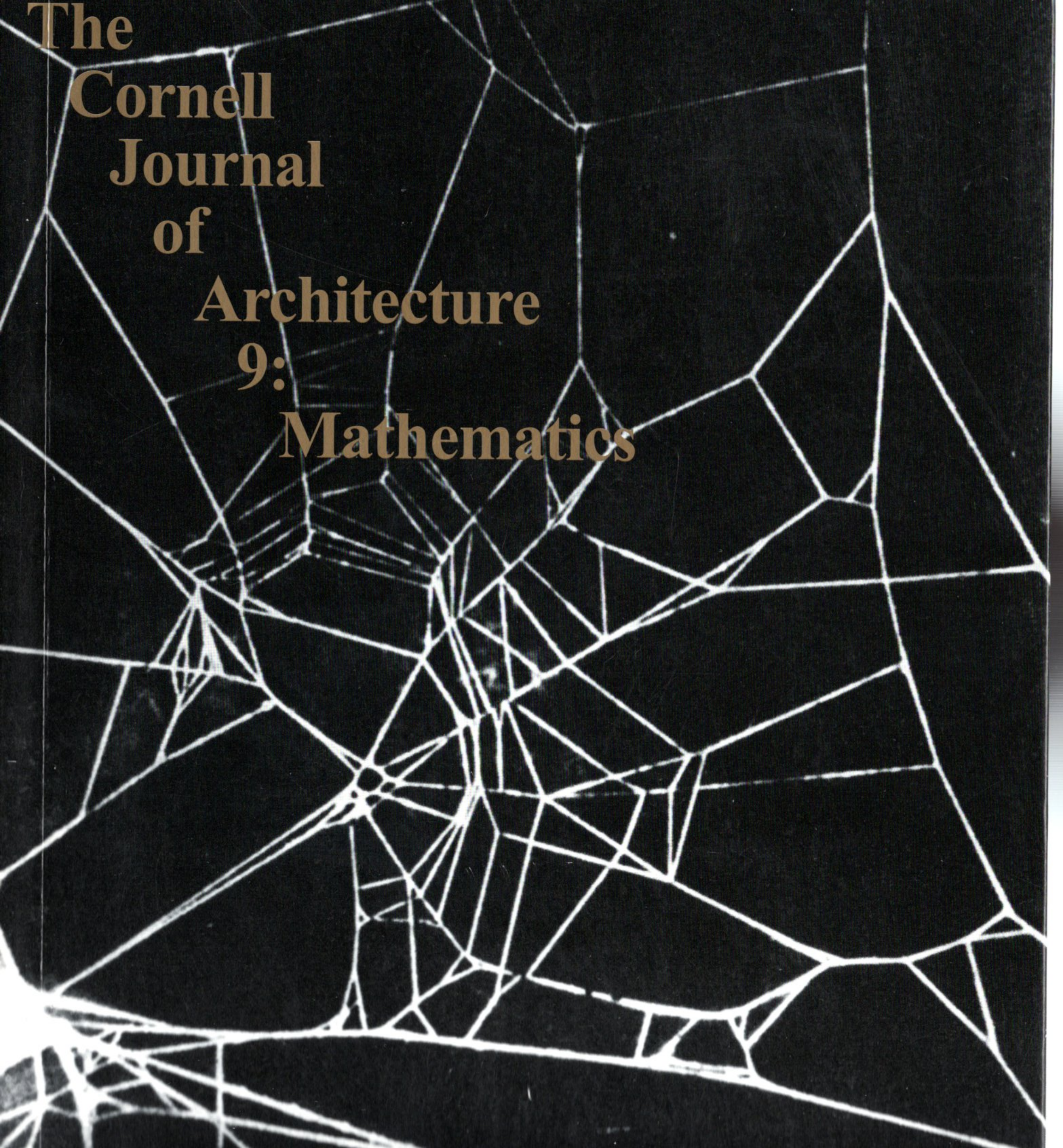
9: Mathematics



From the Ideal to the Uncertain

Cover:

Detail of Rosemarie Trockel, **What it is like to be what you are not**, 1993. The image depicts one of a series of spiderwebs spun under the influence of different drugs, including mescaline, LSD, hashish, and caffeine.

One photogravure from a portfolio of eight photogravures and one photolithograph and one screenprint, composition: 14$\frac{15}{16}$ × 11$\frac{7}{16}$" (38 × 29 cm); sheet: 22$\frac{5}{8}$ × 17$\frac{1}{2}$" (57.5 × 44.5 cm). Publisher: Helga Maria Klosterfelde Edition, Hamburg. Printer: Niels Borch Jensen, Copenhagen. Edition: 9. Carol O. Selle Fund (by exchange). © Rosemarie Trockel, VG Bild-Kunst, Bonn 2012. Courtesy Sprüth Magers Berlin London. © 2012 Artists Rights Society (ARS), New York. Digital Image © The Museum of Modern Art/Licensed by SCALA/Art Resource, NY.

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Digital Remediation

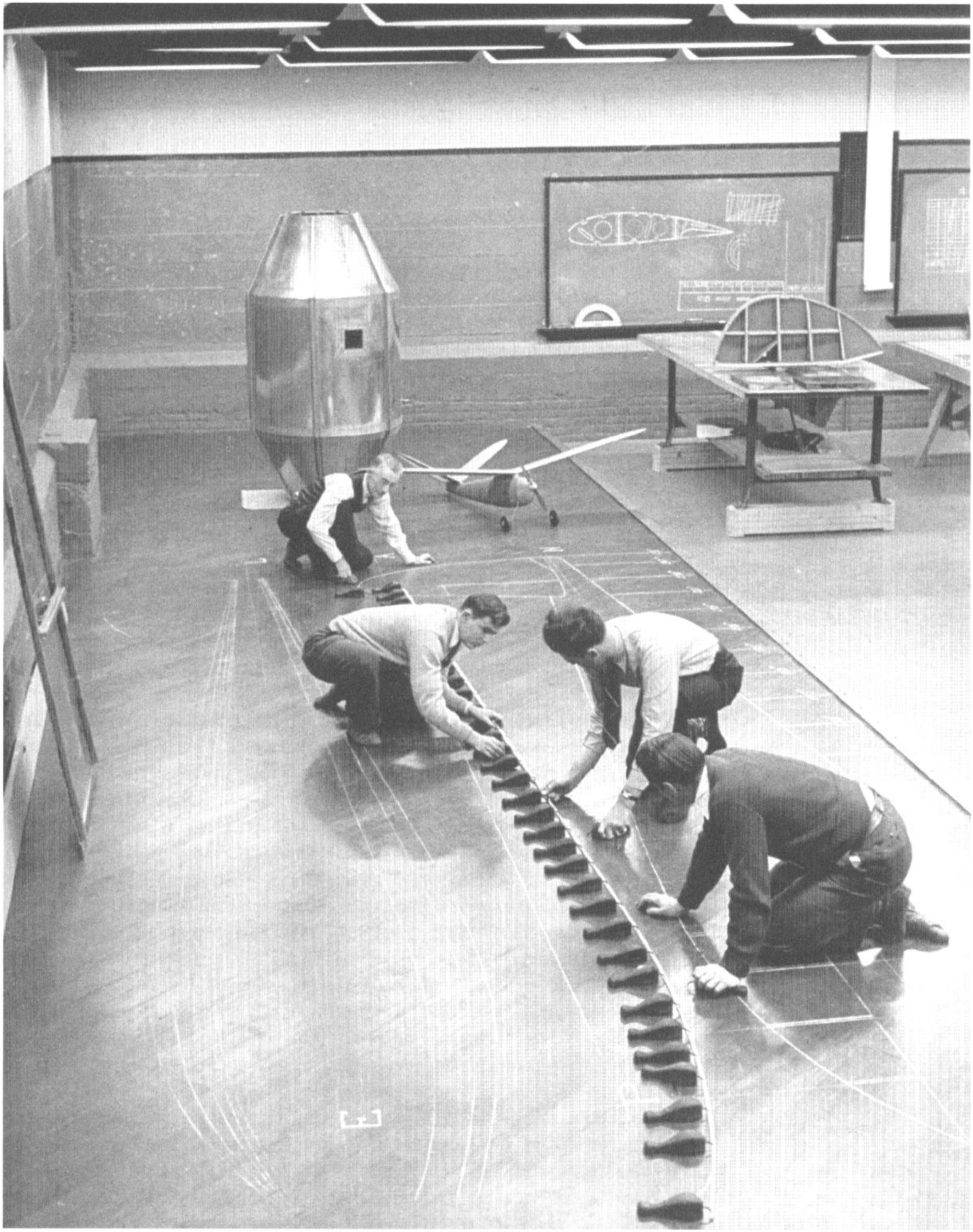
> *The fact that a continuum can be divided in all possible ways, however, is exactly what makes it continuous according to Leibniz. He defines the abstract property of continuity as the potentiality for infinite division in arbitrary ways: there is an arbitrary number of different possible infinite partitions of a continuous whole.*
>
> Richard T. W. Arthur, "Leibniz on Continuity," *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association.*

While geometry is an integral aspect of architectural construction, discourse, and representation, it is never a pure autonomous idea: it is always mediated. If we look at just these three areas, the status of geometry shifts wildly. In construction, geometry involves the measurements that translate drawing to building; in discourse, geometry is a foundation for deductive logic and procedural argument; and in representation, geometry can be the object represented (form), or the mode of representation (projection). Geometry dances beneath, above, and between architecture and its design methodologies. By focusing on geometry's shifting roles in architectural mediation, we can begin to clarify some of the key concerns for architectural representation, and specifically interrogate how digital modeling software is remediating architecture's relation to and with geometry.

Mediation brings with it multiple associations in this context. Most important for the discussion that follows are the associations to Walter Benjamin's essay known in its first two versions as "The Work of Art in the Age of Its Technological Reproduction." In reflecting on ideas of medium and mediation in Benjamin's essay, Tobias Wilke writes, "The medium names the comprehensive force field that links the human sensorium to world and that is constituted in doing so by the interplay between natural (physiological, physical) and historical (social, technological, and aesthetic) factors." This quote in itself is not a bad description of architectural representation, for architecture is not only the art form to master reception by the masses in a state of distraction as Benjamin famously suggests, but it is also the art form whose mode of discourse and practice is fundamentally built on mediation, specifically through the multiple negotiations between abstract geometry, visual expression, and disciplinary interpretation.

One crucial historical shift in architecture's mediation occurs during the Renaissance with the development of scaled, measurable, notated project drawings. These drawings are hybrid objects where the graphic visualization of a building is fused with the regulatory measures of plane geometry. Although geometry and architecture share ancient roots, the desire to tie the image of architectural

Course in Airplane Lofting, Burgard High School, Buffalo, NY, USA, January 1, 1941.



drawing to the rational notations of geometry was one of the crucial developments of early modern architecture. These desires begin the remediation of the medieval stonemason's oral, procedural, full-scale, material construction toward the late Renaissance architect's numerically notated, visually mimetic, proportionately measurable design drawing. Both practices employ the plane geometry descended from Euclid's *Elements*, but the mediation is radically different. First, planar geometric measure allows a precise, pragmatic notation to stand between representation and construction; numeric calculations on the small drawing translate toward material construction in the large building. The significance is that the design control of a building can move away, physically, theoretically, and socially from the building site. Second, geometry becomes fused with the visual image of a building's representation, allowing aesthetic arguments to consider beauty "perceived" as being structured by abstract geometric logic "conceived," meaning that architectural aesthetics could be taught as a code of proportional rules and judged in drawing prior to construction. Third, with the woodblock and the printing press, the mechanically reproduced treatise combined text, representation, and geometry into the same printed page, facilitating an explosion of architectural theory tied to textual discourse, which in turn allows a dissemination of architectural ideas beyond the oral secrecy of the guilds. These treatises required representations to be flat, reduced, and measurable in order that they could be combined with text and numeric notations, and then printed as a plate of coded visual information.

This transformation in the practice and theory of architecture is undeniably of massive importance. But is this transformation the effect of a change in geometry? Or is this transformation the effect of a change in the mediation of the geometries that already exist?

Although orthogonal drawings were known in antiquity, combined orthographic projections begin in earnest with Antonio Sangallo and Albrecht Dürer at the start of the 16th century, are not common in architecture until a century later, and not until Gaspard Monge at the very end of the 18th century do we have a fully resolved system of orthographic projection. If one thinks about these developments as changes in mediation, then the long gestation period for orthographic projection begins to make sense. Each phase strives to achieve what will later be done effortlessly through a change in technology. In many ways, the orthogonal elevation drawings used by early Renaissance architects were not conceived of as projections at all, but more akin to template drawings for the flat measured mappings of elements on an elevation. In this light, the awkwardness of these drawings becomes understandable as they strive to fuse architectural image with the measurements of plane geometry, something more easily achieved through the geometry of orthographic projection. (See Spiral, *Underweysung der Messung,* Albrecht Dürer, on page 22).

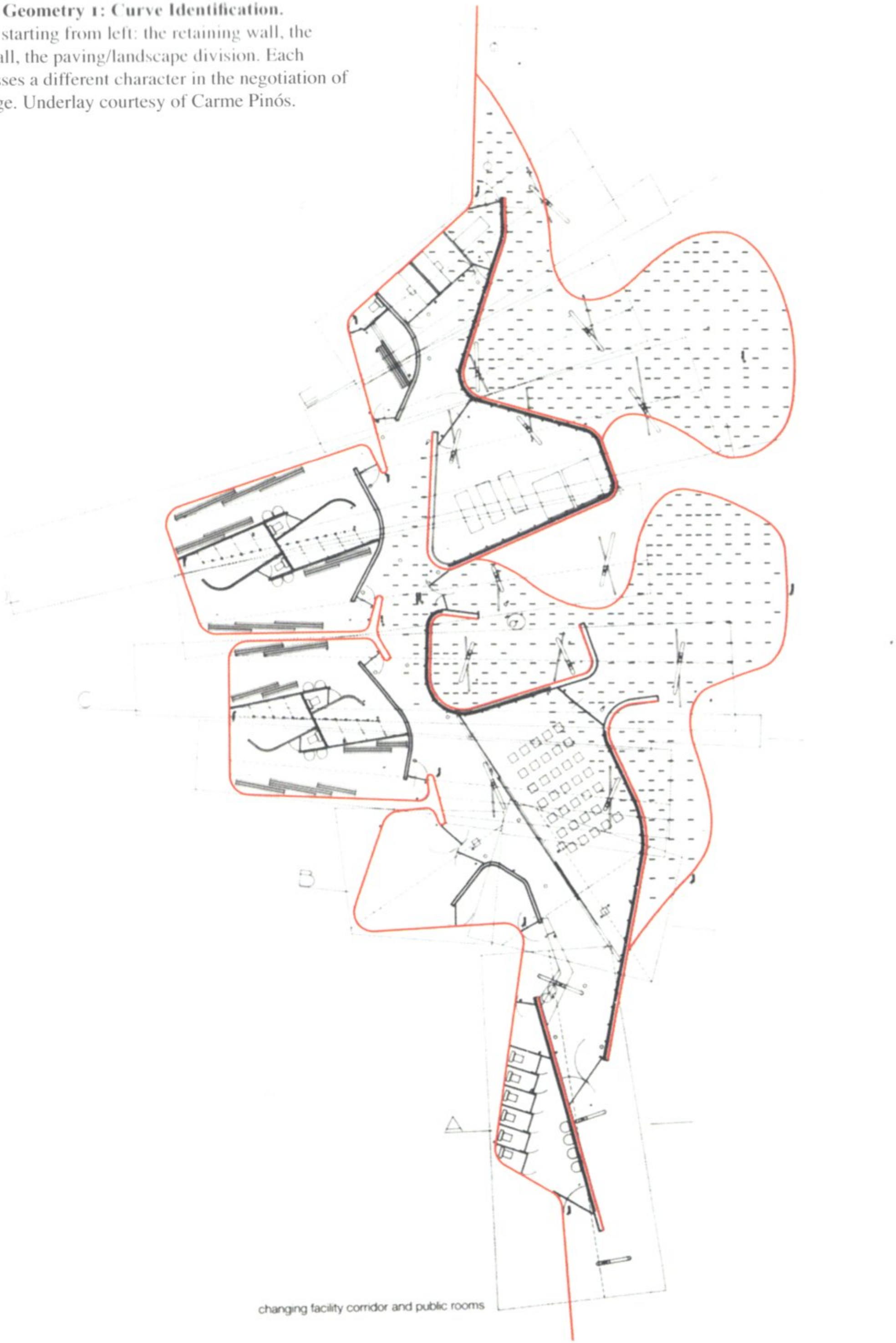
In order to understand the changes instigated by digital representation in architecture, one must be clear about the changing relations between geometry and mediation. The digital modeling software used by architects is structured around plane geometry and projective geometry, measured through coordinate geometry and differential geometry, and represented parametrically. These geometric concepts range from thousands to hundreds of years old. Modeling software does not bring any "new" geometry, but it does radically change the mediation. This change in mediation opens connections to practices, techniques, aesthetics, and concepts that are tangential to much of traditional architecture. It remains to be seen if this remediation will have an equivalent impact as found in previous epochs.

It is often revealing to look at the efforts that an art form goes through prior to a shift in technology. Benjamin's "Artwork" essay provides another salient concept: "The history of every art form has critical periods in which the particular form strains after effects which can be easily achieved only with a changed technical standard—that is to say, in a new art form." If we look at the representations that architects were developing in the decades preceding the advent of the digital model in the 1990s, we find several significant trends. One aspect related to the themes of our present discussion regarding geometry and mediation is suggested by an analysis of the curvature found in the drawings of Enric Miralles and Carme Pinos for the Olympic Archery Range in Barcelona of 1989–92. There are three principle curve lines found in the plan. The first curve is the plan representation of the retaining wall dividing earth from space. The second curve is the primary wall enclosure dividing interior from exterior. The third curve is the landscape line dividing hardscape from softscape. In the analysis shown here, the first act is to measure these curves. The notation is a mapping of translations and rotations regulated through length and angle. These dimensions are all that is required for the full measured description of the first two of the three curves, the retaining wall, and the enclosure wall. But the third curve, the landscape curve, resists the reduction to line and arc and will require a shift in geometric notation. This shift is a move to the notations of tangency, difficult in a manual mediation, fundamental in a digital mediation.

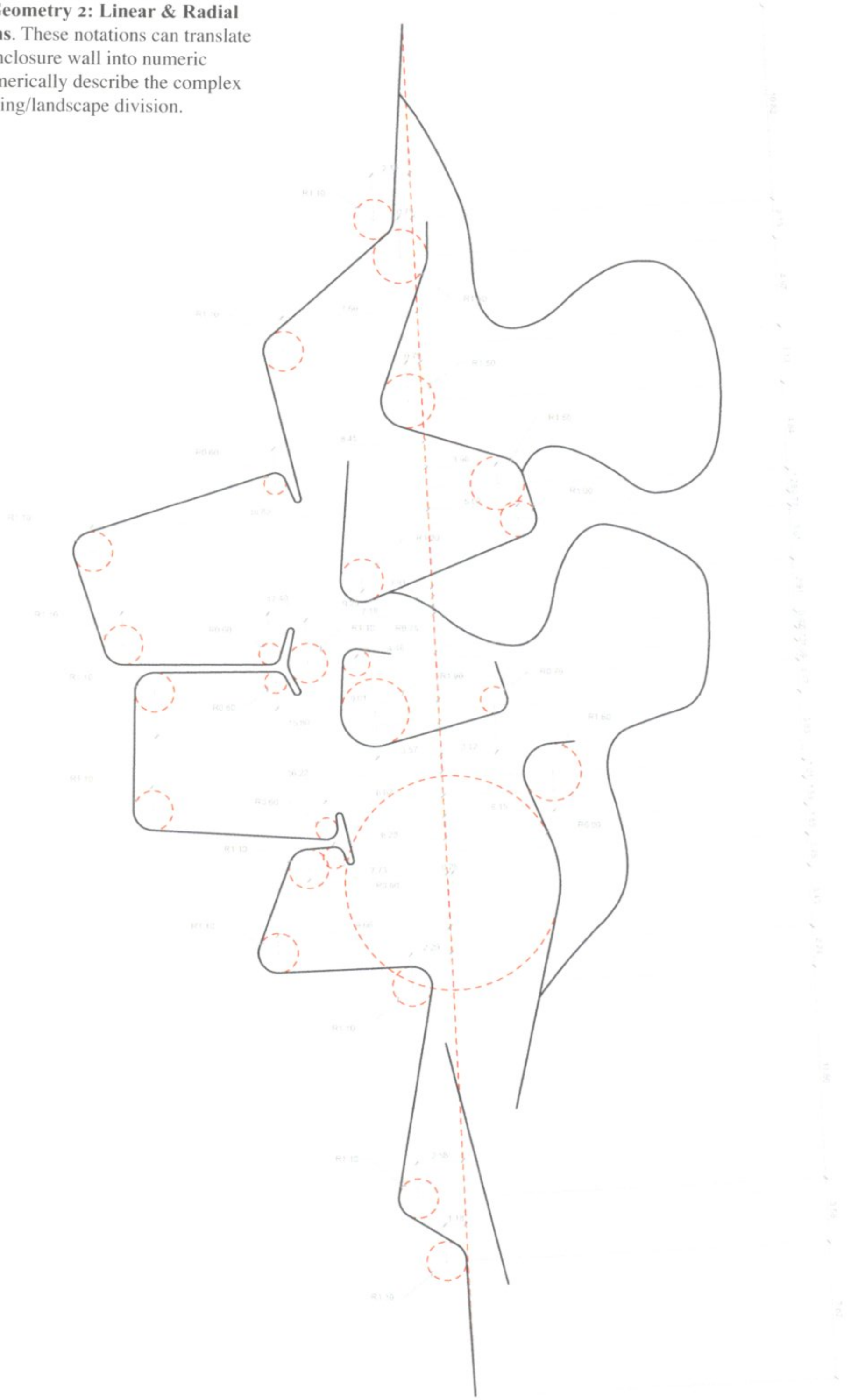
As a generalization, most of the digital modeling software used by architects (Maya, Rhinoceros, 3ds Max, Catia, Generative Components, Revit) can find their origins in a combination of the entertainment industry (film, video games, animation) and the transportation/military industries (ships, airplanes, cars). There are fundamental aspects of digital modeling software more deeply connected to the geometric mediations of automotive, airplane, and shipbuilding traditions than to architecture. Many of the geometric questions in these practices are questions of continuity. The desire is for pieces of surface to join other pieces as smoothly as possible—a necessary condition when surfaces negotiate the flow of water or air. Ship/Plane/Car designers created the need for surfaces that just could not be described with straight lines and arcs, or easily processed through numeric notations. The mathematics for the numeric description of these surfaces came into existence with the application of calculus to geometry. But differential geometry was rather foreign for designers tied closely to artisanal material design practice. It is not until the 1960s that a system of numeric mediation was developed that harnessed the desires of car body designers. This was the digital mediation of curvature that eventually came to be known as non-uniform rational basis splines (NURBS).

In contrast, architecture was successful relatively early on in a numeric resolution through drawing. The design is measured and geometrically resolved in the virtual space of the drawing prior to the construction of the building; construction was to reproduce these drawings precisely, only larger. Because of this, architecture developed different relations to geometry, not only pragmatically but aesthetically as well. It was the hope of Renaissance architects that the beauty of a design could be judged completely from the drawing. Proportion, the relations of part to whole, could be studied in nature, in man, in antiquity, and then taught as "first principles"

Miralles Lines, Curvature Analysis through Graphic Differential Geometry 1: Curve Identification.
Plan Curves starting from left: the retaining wall, the enclosure wall, the paving/landscape division. Each curve possesses a different character in the negotiation of rate of change. Underlay courtesy of Carme Pinós.



Miralles Lines, Curvature Analysis through Graphic Differential Geometry 2: Linear & Radial Measurement Notations. These notations can translate the retaining wall and enclosure wall into numeric notations, but fail to numerically describe the complex curve signifying the paving/landscape division.



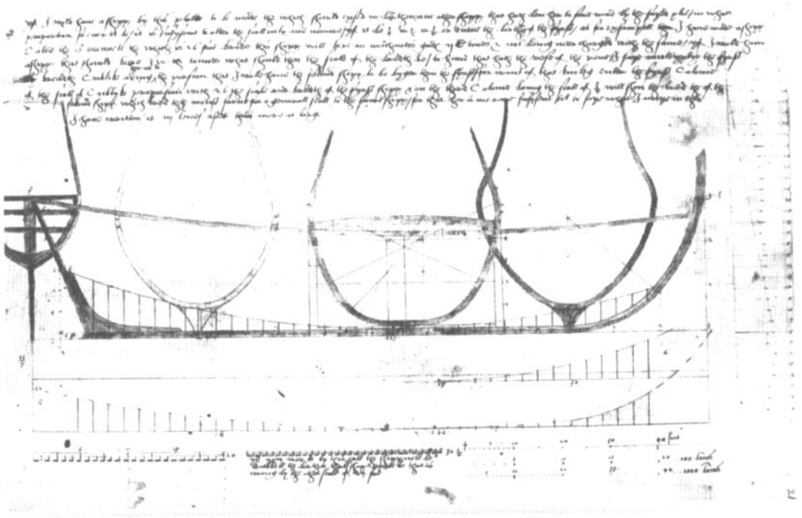
through the mediation of geometrically regulated images. Whether these proportions were understood as natural or conventional, proportion judged visually through a drawing in reference to the abstract ideals provided by geometry proved to be the testing ground for aesthetic beauty. But theories of proportion tied to concepts of ideal beauty were of little value to a ship designer if they produced disjunctions and discontinuities in the surface of the ship's hull. What did matter was the aesthetic that could be seen *and* felt as one surface met another in a smooth continuous manner. This is a visceral sensation, not an idealized linear rationalization. In order to describe and design these surfaces, shipbuilders developed a process that negotiates drawn geometry (lofting), material experimentation (splines and sweeps), and sensory aesthetic judgment (fairing).

During the sixth and seventh centuries, throughout the Mediterranean, there was a shift in building a ship's hull from a "shell-first" toward a "frame-first" construction. This was a conceptual and technological revolution. It meant that the surface of a ship was considered as a sequence of sectional lines transforming along the length of the ship from its midsection toward the front and rear. The ship's hull was thus defined at 90° to its visual elevation through lines that were not edges of the surface but were lines through which the surface was generated. These sectional ribs were constructed as transformations of a master mold or template. Each rib along the length of the boat was a slight modification of the adjacent ribs. This required geometric diagrams that would regulate these transformations along a proportional gradient. This mode of construction also required the sections to be described through profile curves drawn in the plane.

"Lofting" loosely describes a group of techniques that generate a surface description through sectional profiles. The minimum situation is two guide curves, which results in a ruled surface. These straight lines join the two curves at points of common tangency, described by where a flat plane would touch both curves.

(L) **Ship Draught from Matthew Baker's Manuscript**, 1586. From Peter Jeffrey Booker, *A History of Engineering Drawing* (London: Chatto & Windus, 1963).

(R) **20th-Century Ship Drawing** from Richard M. Van Gaasbeek, *A Practical Course in Wooden Boat and Ship Building* (Chicago, IL: Friedrick J. Drake, 1918).



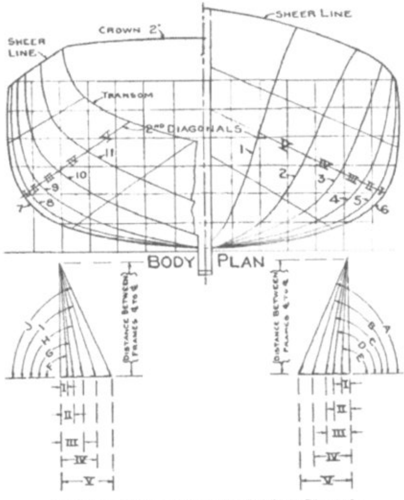


Figure 24.—Method of Developing Bevels on Diagonals.

The joining lines in turn can be sectioned to produce intermediate guide curves, and the process recursively repeats itself. By the 17th century, lofting developed into the use of three sets of interrelated sectional curves describing the surface of the ship's hull in three perpendicular directions: body plan (front), sheer plan (side), and half-breadth plan (top). These techniques are some of the most sophisticated forerunners of Mongean descriptive geometry. Once the hull curves are interrelated through the three views, a complex surface of double curvature can be lofted as a network of contour lines.

Lofting mediates a three-stage translation: drawn geometric diagrams, full-scale fair splines, and mold templates for hull construction. The curves are never completely measured on the small-scale drawing in the sense that architects understand plans; they are constantly negotiated as graphic lines and material lines, at reduced scale and full scale, and only when all the interrelations of these networks prove to be fair are the templates constructed. The curve becomes the primary representational element not as the boundary edge, but as a notation of the edgeless condition interior to a surface of freeform curvature, a contour.

Fairing is a process of testing these lofted lines for continuity. This is done visually by looking down the length of the line checking for kinks, and tactilely by running a hand along a material line or "spline" to check for smoothness. Fairing requires sensory judgment of geometric and material relations, and the ability to manipulate a curve locally to be smooth and continuous. As the ship lines are drawn at full scale on the loft floor (the floor was elevated to be flat and level for inscription of the full-scale lines and the construction of templates; this elevated floor is what gives "lofting" its name), the curves are marked out by long strips of flexible wood or metal called "splines," and held in place by weights sometimes referred to as "ducks."

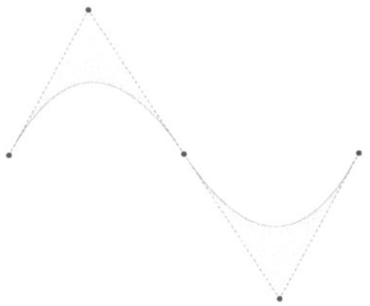
A spline is a material line, using the properties of bending in the material and moments of force constraining the spline into various configurations. These weights that hold the splines in place allow local variations of the curve, in order to ensure smoothness and continuity of the curves as they vary in complex manners across the hull. Manipulating a related series of curves controls the continuity of the surface. These manipulations are performed point by point, weight by weight, with subtle local variations based continually on tactile and visual sensations of fairness.

A physical template cut from a fair curve could become a guide for future curves. This is how ship templates and eventually French curves came to be developed. These are templates from material constructions that would come back into a drawing to guide the stylization of graphic lines. These templates could also be run along another guide-rail template carving out clay or sand as a complex doubly curved surface; this technique is called a "sweep." Often shipbuilding curve templates would be referred to solely as sweeps. It should be noted that many of these terms touched on above are also used in modeling software. Terms such as *lofting, sweeping, patching, splines*, and *fairing* have become digital algorithms, but all of these techniques were once design mediations moving between the physical and abstract, the material and geometric, the template and the line, the hand and the eye, in order to resolve the continuities of surface.

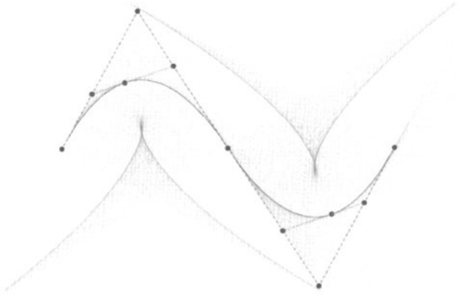
The major steps taken to develop a description appropriate for computer-driven computation came from the automotive design industry of the 1960s. The question that faced these designers and engineers was how to provide a numeric description of free-form surfaces with an interface intuitive enough for designers to feel that they could handle the subtle variations of curvature that functional and aesthetic performance required. The need for a numeric definition of these curves was driven by the rise of numeric control in fabrication. In order for any digital fabrication system to handle a construction, it had to be described through numbers.

Two key players in this development were Paul de Casteljau and Pierre Bézier, who worked for the French automobile companies Citroen and Renault, respectively. Both engineers developed a method for a parametric representation that could allow a direct user interface in the construction and manipulation of free-form curves and surfaces. The algorithm goes under the name of De Casteljau, while the curves are called Bézier, due to publishing time sequences. Both engineers were attempting to solve very pragmatic problems. In the description of surface curvature at the time, auto body designers would use templates, French curves, and sweeps to mold clay, sand, or other materials in the desired shape. These templates could then be used to transfer lines of the car body to a drawing, then scaled up to have jigs and molds fabricated. This was more or less traditional shipbuilding technique and technology, not radically different from the way free-form surfaces were described during the 15th century. But in the mid-20th century, the automotive and aircraft industry were running into new problems as every other component in the assembly was increasing its precision. The surface pieces were not always able to keep up with diminishing tolerances and, importantly, milling and fabrication machines were beginning to appear that could be Computer Numerical Controlled. Their task required a new mediation of the geometry. Paul de Casteljau began with an old construction for a parabola called a "thread construction." Its simplest example is two rigid sticks with strings connecting equal divisions on both sticks, the sequence flipped so that first is connected to last. This simple construction defines a parabolic curve, but is also an example of a recursive algorithm of repeated linear interpolation.

Thread Construction of Parabolic Curve. Simplest example of de Casteljau's algorithm, Michael Young, 2011.

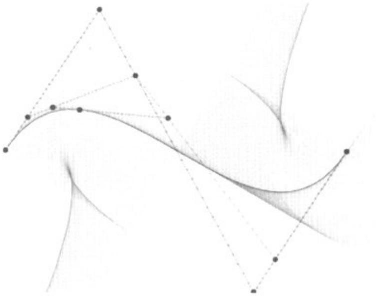


Degree 2 NURBS Curve, with lines tangent and normal. This example shows the stitching of the 2° two Bézier curves at the midpoint of the control polygon as a basis spline. A single iteration of De Casteljau's algorithm has been graphically represented. Michael Young, 2011.

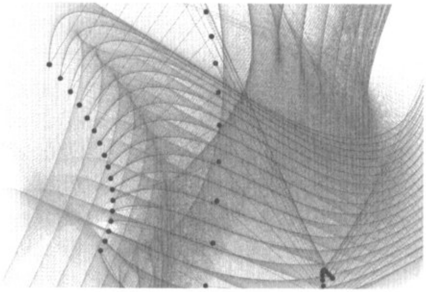


The sticks will become the control polygon, the ends become the control points, and the strings become the vectors tangent to a curve. This construction is at the heart of Bézier curves, which join together to form basis splines, which are rationalized to create NURBS modeling software, the most common way to define curvature and surface in a digital environment.

Non-uniform rational basis splines are a calculus-based parametric mediation of curvature. Tangent vectors are the primary elements of curve measurement, and Gaussian surface analysis allows the measurement of surface curvature through the differentiation of vectors normal to a surface.



Degree 3 NURBS Curve, with lines tangent and normal. This example shows a single, degree three Bézier curve. A single iteration of De Casteljau's algorithm has been graphically represented. Michael Young, 2011.



Degree 3 NURBS Curve Field, with lines tangent and normal. This example shows the transformation of a curve through several stages, generating a surface, notated as a field of vectors. Michael Young, 2011.

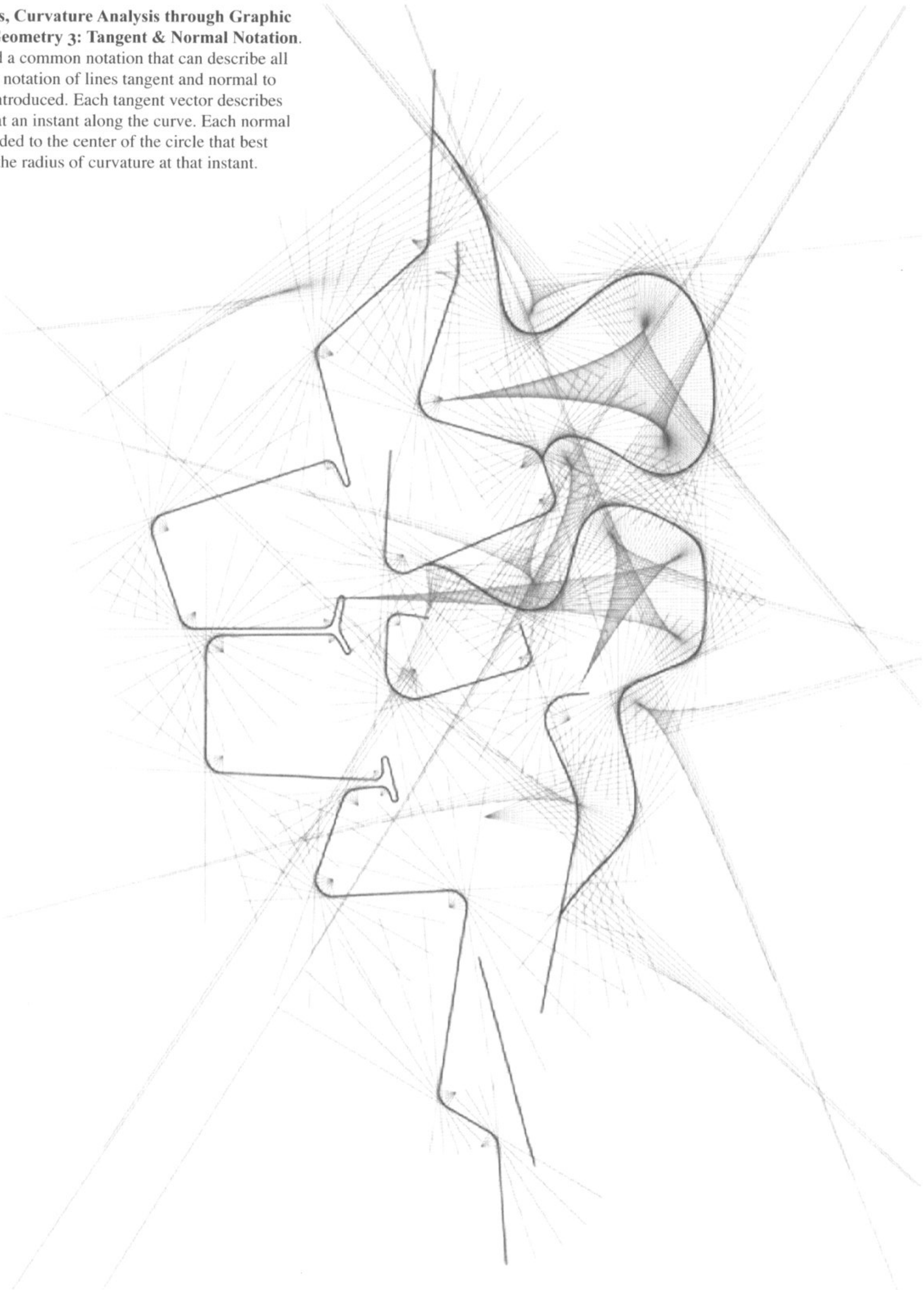
The mathematical representation of curves and surfaces in a digital environment is parametric. Much has been made of parametric representation in contemporary architectural discussions, both practice-based arguments with BIM software and design-based apologies, but it is necessary to remember its roots and why design software is for the most part parametric. A parametric representation allows numeric definition without explicit dependency to the *xyz* coordinate axes that define the global digital space. Instead, a parameter—time, for example—is introduced and the spatial equations are redefined to depend on the domain of this new parameter. This is crucial for digital modeling since the design must be able to be transformed as an object in space. If digital entities were defined through explicit equations, every time you moved an object it would change because it would be dependent on the global *xyz* values. Parametric representation also defines an entity through a bounded domain. This bounding of the parameters means that every curve has a start and an end. The curve's vectors have a directionality informing the sequence with which it is computed. Furthermore, the parameters that the surface or curve equations depend on can be associated to more than one entity. The manipulation of one variable thus begins to affect a large number of associated entities. These parametric associations can build a surface, or can tie together multiple objects as an aggregate of relations.

The mediation for the control of geometric transformations is fundamentally different between the analog and digital. In an analog regulation, the instruments of a straightedge or compass leave a residue of their action. These traces accumulate as a design develops; layers of translucent paper are often used to sort out the sequence. In a digital environment, there is no trace or index of the transformations. To visualize a previous state, the designer has to undo the operation, or make several copies of each state. But as the graphic index is lost, there are new qualities that emerge from the digital mediation of transformation. Transformations regulated parametrically represent a range of possible states available along a vector; each instance is a simulation. Most software can also nest several transformations into a single operation to allow what appears to be deformation. Twists, bends, tapers, and stretches are examples of this. There is not really any new transformation here, but a combination of simpler transformations associated parametrically. It begins to appear that you can deform a surface. This is a major conceptual change for the mediation of geometry. Within a traditional mediation of geometry, a deformation is an immeasurable, intuitive act that typically involves a physical exchange between matter and force. It is important to remember that splines used in shipbuilding are deformable lines. There is a deep connection here in the desire to manipulate a curve or a surface in a free, deformable manner with tactile and muscular sensations of pressure altering the material. The movement of control points alters the net of control polygons, which alters the computation of each associated curve, and thus any associated surface can be locally, internally deformed in what appears to be real time. This is the kind of interface that a car body stylist or film animator desires; control points, tangent threads, and normal vectors would just get in the way of visualizing the object under design. Similarly, the lack of residual index left by the parametric representation of design transformations is preferred by these industries; they are after the associations and affects of a final design image and do not require a procedural logic with which to justify their actions.

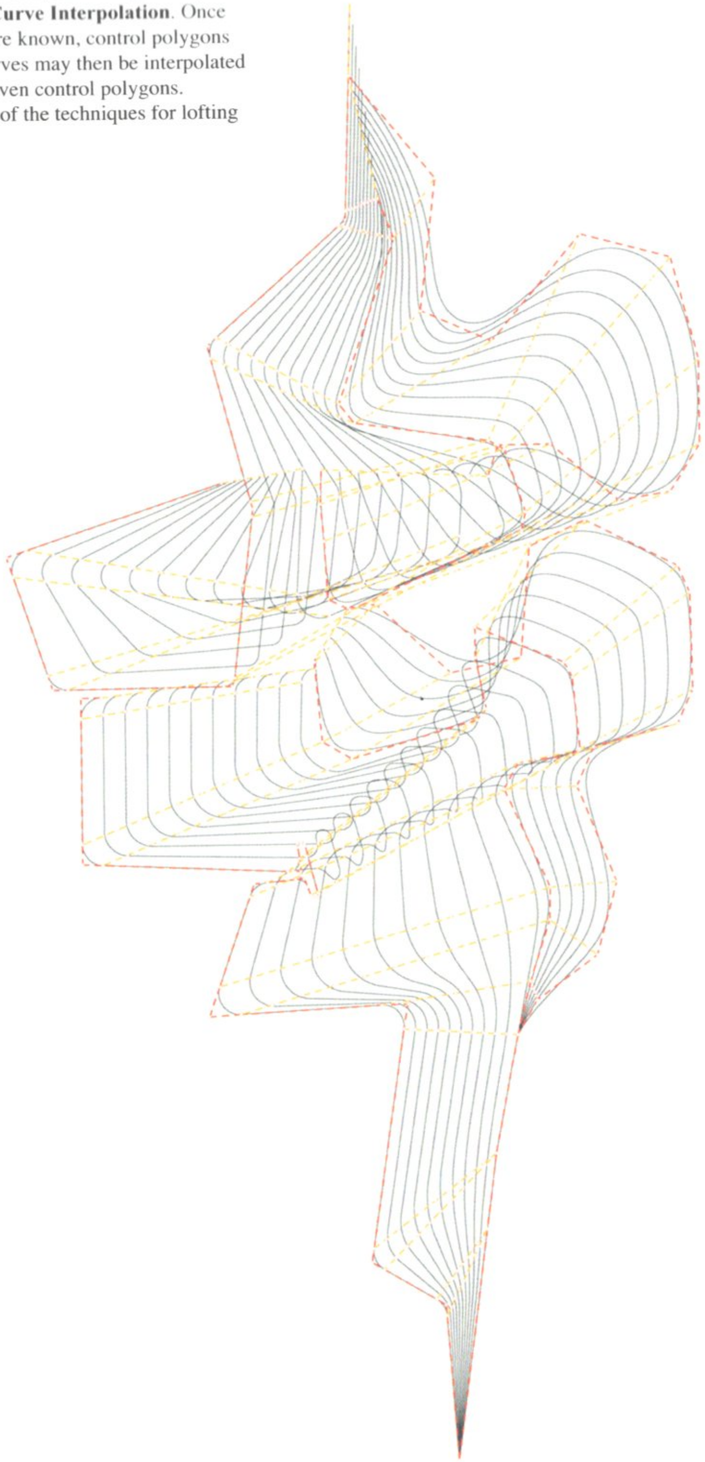
It is now time to return to the analysis of the Miralles and Pinos plan. As will be recalled, the complex curvature of the free-form landscape curve could not be rationalized into arcs and straight lines. The discussion of shipbuilding curves and tangent notations provide us with a compelling option for describing this curve. Viewing curvature through the lens of calculus and differential geometry alters the designer's understanding of a curve. A single tangent notation gives a graphic description of the rate of change of curvature, and as a collective of tangents, how fast and in what ways these rates change. The line normal to the curve also becomes a unique graphic device as a vector showing the instantaneous radius of curvature. Lines tangent and normal can become part of a graphic notation relating conceptual and aesthetic aspects of curvature. Curves loose a singular signification due to particular shape, and instead gain the qualitative character of curvature. The control points define an envelope for the curve to develop. It is within this envelope that the curve will be approximated through subdivisions of a control polygon. You do not draw the curve; you sequentially set spatial vectors within which the curve will be simulated. The first two points set a direction of tangency for the start of the curve; the last two points set the ending tangent vector. These tangents establish the control polygons, which like a marionette, allows transformations of curves at a distance. Once the three Archery Range curves are described through the notations of tangency, their control polygons can be constructed. These control polygons can be linearly interpolated between themselves to build a field of curve instances between states. This technique is the digital descendent of shipbuilding lofting.

If curves are measured as the locus of a moving line tangent to the curve, the curve is dissolved into variations of vectors, or notations of force. If surfaces are measured as the regulated transformation of curves, then the surface dissolves into a field of vectors. The lines tangent give a vector notation of movement along the curve, and in a surface, the lines normal give a vector notation between curves as the surface deforms. Geometrically, the digital construction is explicitly notated. But, aesthetically, we no longer see surfaces or lines, but fields of forces, their movements structuring organizations of density, accumulation, and direction. Surfaces become notational extensions into fields of variable intensity.

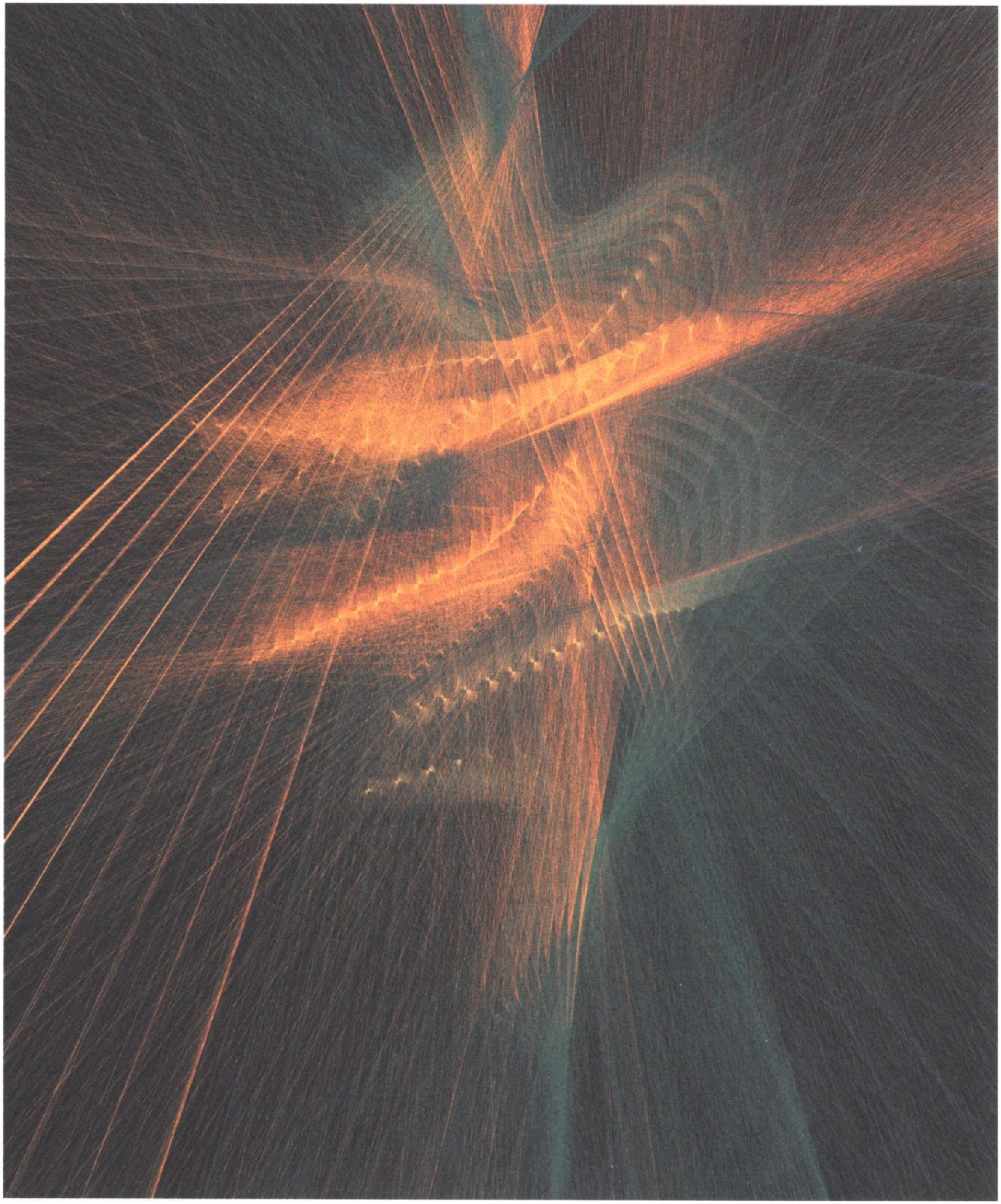
Miralles Lines, Curvature Analysis through Graphic Differential Geometry 3: Tangent & Normal Notation. In order to find a common notation that can describe all three curves, a notation of lines tangent and normal to the curves is introduced. Each tangent vector describes the trajectory at an instant along the curve. Each normal vector is extended to the center of the circle that best approximates the radius of curvature at that instant.



Miralles Lines, Curvature Analysis through Graphic Differential Geometry 4: Curve Interpolation. Once the tangents of each curve are known, control polygons can be constructed. New curves may then be interpolated between each of the three given control polygons. This is a graphic equivalent of the techniques for lofting a surface.



Miralles Lines, Curvature Analysis Through Graphic Differential Geometry 5: Vector Fields of Tangent and Normal Notations. Density, Length, and Color are variable along a gradient, these are the only variables. The gradient variations of these quantities require an aesthetic response from the designer to evaluate and control the emerging qualities.



As architecture re-mediates it relations to representation through digital technologies, it becomes imperative to understand the nature of this change. Not only to be able to guide the software toward specific architectural concerns, but to be able to understand the conceptual and aesthetic desires that are latent in the mediation itself: and in this balance begin to experiment with other possibilities that the software may open. The lines tangent and normal to a curve are in many ways the graphic notations of the digital tectonic building a curve. To work through their representation is to pull in a deep historical discourse in which architectural notation seeks to record and reveal the acts of construction through acts of representation and vice versa. But of equal importance here in the final drawing of the analysis of the Archery Range plan are the aesthetic concerns that begin to accumulate around the variable intensities of vector notations. These aesthetic affects are not new; they are in fact very painterly and in many ways best understood in reference to late 19th-century aesthetic discourse. What is novel is the mediation of painterly atmospheric sensations through differential geometry. To pursue the aesthetic as well as the pragmatic in any technology is to begin to use that system for what it truly is: mediation.

Endnotes

1 Tobias Wilke, "Tacti(ca)lity Reclaimed: Benjamin's Medium, the Avant-Garde, and the Politics of the Senses," *Grey Room* 39 ((Spring 2010): 40.

2 Walter Benjamin, "The Work of Art in the Age of Its Technological Reproducibility," *Grey Room* 39 (Spring 2010): 33.

3 Filippo Camerota, "Renaissance Descriptive Geometry: The Codification of Drawing Methods," in *Picturing Machines: 1400–1700*, Wolfgang Lefevre (ed.) (Cambridge, MA: MIT Press, 2004), 175–176.

4 Wolfgang Lefevre, "The Emergence of Combined Orthographic Projections," in *Picturing Machines: 1400–1700*, Wolfgang Lefevre (ed.) (Cambridge, MA: MIT Press, 2004), 210–211.

5 James Ackerman, "The Origins of Architectural Drawing in the Middle Ages and Renaissance," in *Origins, Imitation, Conventions* (Cambridge, MA: MIT Press, 2002), 49–53.

6 Mario Carpo, "The Making of the Typographical Architect," *Paper Palaces* (New Haven, CT: Yale University Press, 1998), 159–160.

7 Both the medieval use of plane geometry as practical trade-based constructions and the Renaissance theories of proportion derived from Vitruvius are ultimately examples of the first written discourse on proportion known from classical antiquity; Book V from Euclid's *Elements*.

8 Mario Carpo, *The Alphabet and the Algorithm* (Cambridge, MA: MIT Press, 2011), 21–22.

9 Dan Pedoe, *Geometry and the Visual Arts* (New York: Dover Publishing, 1976), 102–103.

10 Mario Carpo, *Architecture in the Age of Printing* (Cambridge, MA: MIT Press, 2001), 11–14.

11 Bruno Latour, "Drawing Things Together," in *Representation in Scientific Practice*, Michael Lynch and Steve Woolgar (eds.) (Cambridge, MA: MIT Press, 1990), 46.

12 "But the last advantage is the greatest. The two-dimensional character of inscriptions allow them to merge with geometry. As we saw for perspective, space on paper can be made continuous with three-dimensional space. The result is that we can work on paper with rulers and numbers, but still manipulate three-dimensional objects 'out there' (Ivins, 1973). Better still, because of this optical consistency, everything, no matter where it comes from, can be converted into diagrams and numbers, and combination of numbers and tables can be used which are still easier to handle than words or silhouettes (Dagognet, 1973). You cannot measure the sun, but you can measure a photograph of the sun with a ruler. Then the number of centimeters read can easily migrate through different scales, and provide solar masses for completely different object." 46.

13 Wolfgang Lefevre, "The Emergence of Combined Orthographic Projections," in *Picturing Machines: 1400–1700*, Wolfgang Lefevre (ed.) (Cambridge, MA: MIT Press, 2004), 210–211.

14 Walter Benjamin, "The Work of Art in the Age of Its Technological Reproducibility," *Grey Room* 39 (Spring 2010): 27.

15 Robin Evans, *The Projective Cast: Architecture and Its Three Geometries* (Cambridge, MA: MIT Press, 1995), 113–116.

16 Walter Benjamin, "The Work of Art in the Age of Its Technological Reproducibility," *Grey Room* 39 (Spring 2010): 31.

17 In an effort to focus on the mediation of geometry, this paper will address only the latter of these two. A similar effort would be valuable in looking at the meshing, rendering, and lighting engines that come from entertainment design practices.

18 Gerald Farin, *Curves and Surfaces for CAGD* (San Francisco, CA: Morgan Kaufmann, 2002), 1–2.

19 Gerald Farin, *Curves and Surfaces for CAGD* (San Francisco, CA: Morgan Kaufmann, 2002), 1–2.

20 Mario Carpo, *The Alphabet and the Algorithm* (Cambridge, MA: MIT Press, 2011): 70.

21 Caroline van Eck, "Verbal and Visual Abstraction: The Role of Pictorial Techniques of Representation

in Renaissance Architectural Theory," in *The Built Surface, vol. 1*, Christy Anderson (ed.) (Hants, UK: Ashgate Publishing, 2002), 167–169.

22 Indra Kagis McEwen, "On Claude Perrault: Modernizing Vitruvius," in *Paper Palaces* (New Haven, CT: Yale University Press, 1998), 324–326.

23 Eric Rieth, "To Design and to Build Mediaeval Ships (Fifth to Fifteenth Centuries)—The Application of Knowledge Held in Common with Civil Architecture, or in Isolation?" in *History of Science and Medicine Library, Volume 11: Creating Shapes in Civil and Navel Architecture*, Horst Nowacki and Wolfgang Lefevre (eds.) (Leiden, Netherlands: Koninklijke Brill, 2009), 122.

24 Horst Nowacki, "Shape Creation Knowledge in Civil and Navel Architecture," in *History of Science and Medicine Library, Volume 11: Creating Shapes in Civil and Navel Architecture*, Horst Nowacki and Wolfgang Lefevre (eds.) (Leiden, Netherlands: Koninklijke Brill, 2009), 31–33.

25 Gerald Farin, *Curves and Surfaces for CAGD* (San Francisco, CA: Morgan Kaufmann, 2002), 441.

26 Howard Thrasher, *Aircraft Lofting and Template Layout* (San Francisco, CA: Aviation Press, 1942), 125–128.

27 Peter Jeffrey Booker, *A History of Engineering Drawing* (London, UK: Chatto & Windus, 1963), 68–70.

28 William Nelson, *Airplane Lofting* (New York: McGraw-Hill, 1941), 82.

29 Richard M. Van Gaasbeek, *A Practical Course in Wooden Boat and Ship Building* (Chicago, IL: Friedrick J. Drake, 1918), 179–180.

30 Edward L. Attwood, *A Text-Book of Laying Off or the Geometry of Shipbuilding* (London: Longmans, Green & Co, 1918), 16, 19.

31 Richard M. Van Gaasbeek, *A Practical Course in Wooden Boat and Ship Building* (Chicago, IL: Friedrick J. Drake, 1918), 179–180.

32 Gerald Farin, *Curves and Surfaces for CAGD* (San Francisco, CA: Morgan Kaufmann, 2002), 2–11.

33 Helmut Pottman, Andreas Asperl, Michael Hofer, and Axel Kilian, *Architectural Geometry* (Exton, PA: Bentley Institute Press, 2007), 259.

34 Gerald Farin, *Curves and Surfaces for CAGD* (San Francisco, CA: Morgan Kaufmann, 2002), 8–9.

35 Ibid., 43–45.

36 NURBS: Non-Uniform Rational Basis Splines.

37 Non-Uniform: Knots, the joins between Basis or B-Spline curves, can be uniformly or non-uniformly spaced along the length of a curve.

38 Rational: Mathematically defined 3-space polynomial equations for all curves. This allows each control point to be weighted differently than the others, which is achieved through central projection. Conics such as hyperbolas, ellipses, and circles are thus definable.

39 Basis Splines: Several Bézier curves joined together in uniform knots, a piecemeal curve. The curvature is smooth at the knot, as the end of one curve shares tangency and vector length with the start of the next, creating parametric continuity.

40 Bézier Curve: Parametric method of determining a smooth curve within a control polygon through linear interpolation. Control polygon is divided in equal ratios, lines join the divisions, and these are further divided along the same ratio; this sequence is iterated according to degree of curvature to determine a point on the curve. The collection of all points and their corresponding tangents determine the complex curve. Named for Pierre Bézier, who published it first, it uses Paul de Casteljau's algorithm.

41 Curvature Degree: Refers to the highest exponential in the curve's equation. Degree 1 = Straight Line, Degree 2 = Quadratic Curve (Conic), Degree 3 = Cubic Curve, and so on. The degree of curvature cannot be higher than the sides of the control polygon determined by the control points of a curve.

42 David Celento, "Innovate or Perish: New Technologies and Architecture's Future," in *Fabricating Architecture*, Robert Corser (ed.) (New York: Princeton Architectural Press, 2010), 63–65.

43 Patrik Schumacher, "Parametricism and the Autopoiesis of Architecture," *LOG* 21: 63–79.

44 It should be said that the author has deep reservations about many of the claims made in defense of parametric tools as a design style. This paper does not have space for this argument and will have to wait to be pursued.

45 David F. Rogers, *An Introduction to NURBS* (San Francisco, CA: Morgan Kaufmann, 2001), 2–4.

46 Ibid., 156–157.

47 Helmut Pottman, Andreas Asperl, Michael Hofer, and Axel Kilian, *Architectural Geometry* (Exton, PA: Bentley Institute Press, 2007), 453–468.

48 Gilles Deleuze, *Francis Bacon: The Logic of Sensation* (Minneapolis, MI: University of Minnesota Press, 1981), 50.

49 "These are two very different categories. The transformation of form can be abstract or dynamic. But deformation is always bodily, and it is static, it happens at one place; it subordinates movement to force, but it also subordinates the abstract to the Figure."

50 Bernard Cache, *Earth Moves* (Cambridge, MA: MIT Press, 1995), 49–51.

51 David F. Rogers, *An Introduction to NURBS* (San Francisco, CA: Morgan Kaufmann, 2001), 46.

52 Gerald Farin, *NURBS From Projective Geometry to Practical Use* (Natick, MA: AK Peters, 1999), 110–112 and 159–161.

53 Heinrich Wolfflin, *Renaissance and Baroque* (London: Fontana Library, [1888] 1964), 29–37: "The contour is quite annihilated, and the continuous, static lines of the old style are replaced by and indistinct and gradually fading boundary area," 31. "The painterly style gives the illusion of physical relief, and the different objects seem to project or recede in space," 31. Alois Riegl, *Historical Grammar of the Visual Arts* (New York: Zone Books, [1898] 2004), 129. "Once movement was deemed acceptable, transitory and accidental qualities entered art," 129. Wilhelm Worringer, *Abstraction and Empathy* (Chicago: Ivan R. Dee, [1908] 1997), 21. "A crucial consequence of this artistic volition was, on the one hand, the approximation of the representation to a plane, and on the other, strict suppression of the representation of space and exclusive rendering of a single form," 21.